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Electrical Waves and the Behavior of Long-Distance Transmission Lines

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Electrical Waves and the Behavior of Long-Distance Transmission in Lines.

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[This paper develops Maxwell's mechanical conceptions of electromagnetism and applies them in a very simple and direct way to the discussion of electromagnetic waves.

This fundamental discussion of electromagnetic waves is followed by a discussion of wave distortion on telephone lines. The effect of line resistance and line leakage upon wave distortion is explained and a very elementary statement is given of the reduction of line losses (in pure wave transmission) by the increase of line inductance.—The Editor.]

In the discussion of any class of physical phenomena the choice lies between two methods, one making use of conceptions, generally mechanical conceptions, and the other employing elaborate mathematical developments, usually based upon differential equations. The present purpose is to describe in popular but precise terms, the phenomena of electric waves and especially to give an intelligible account of electric wave distortion. For this purpose the purely mathematical method is utterly hopeless and unfortunately the conceptual method is nearly hopeless also for the reason that the conceptions which one must employ in describing electromagnetic actions are not only unfamiliar but they are, to a great extent, antagonistic to those more or less indefinite but extremely persistent mechanical notions which underlie nearly every one's idea of electricity.

In this discussion I shall make use of Maxwell's cellular theory of the ether and I assure you that you need not concern yourselves with the limitations of this theory. It is more than adequate to our present purpose. I shall devote a large part of the time at my disposal in developing the conceptions of electromagnetic action which are based upon Maxwell's idea of a cellular ether, and in this discussion you are to understand that I am not contending for the truth of these conceptions, I am only insisting on their utility.

Fundamental Conception.—The ether is to be considered as built up of very small cells of two kinds, positive and negative, in such a way that only unlike cells are in contact. These cells are imagined to be so connected, where they are in contact. that if a cell be turned while the adjacent cells are kept stationary, then a torque, due to elastic reaction of adjoining cells, is brought to bear upon the turned cell, which tends to right it, and which is proportional to the angle turned.

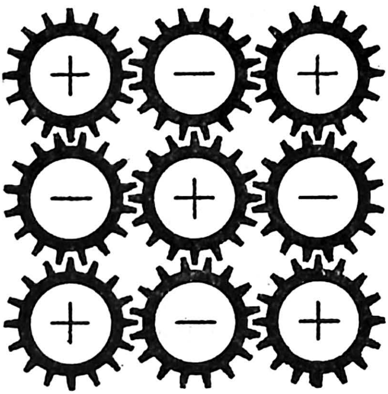

Fig. 1.

For example, the ether cells may be thought of as small cog wheels with rubber teeth, positive cells and negative cells gearing into each other, as shown in Fig. 1. In subsequent figures these cog wheels are represented by plain circles.

Conception of the Magnetic Field.—The ether cells at a point in a magnetic field are to be thought of as rotating about axes which are parallel to the direction of the field at the point, the angular velocitiy of the cells being proportional to the intensity of the field at the point. The positive cells rotate in the direction in which a right-handed screw would be turned that it might move in the direction of the field, and the negative cells

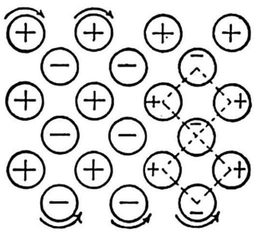

Fig. 2.

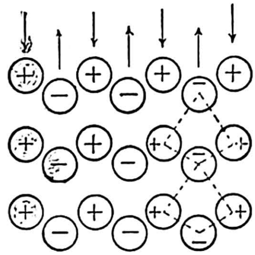

Fig. 3.

rotate in the opposite direction. This opposite rotation of positive and negative cells is mechanically possible since only unlike cells are geared together.

This rotatory motion of the ether cells is indicated in Fig. 2, which represents a magnetic field perpendicular to the plane of the paper, and directed away from the reader; all the positive cells rotate clockwise, and the negative cells counter-clockwise. The kinetic energy per unit volume in such a system of rotating cells is proportional to the square of the angular velocity, which is consistent with the fact that the energy (kinetic) per unit volume in a magnetic field is proportional to the square of the intensity of the field.

Conception of the Electric Field.—The positive ether cells at a point in an electric field are to be thought of as displaced in the direction of the field, while the negative cells are displaced in the opposite direction; this displacement being proportional to the field intensity. Thus Fig. 3 represents the case in which the positive cells have been displaced towards the bottom of the page relatively to the negative cells. Fig. 4 represents two

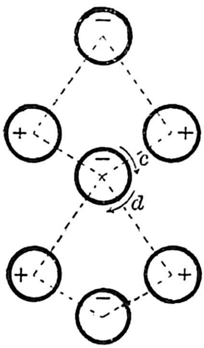

Fig. 4.

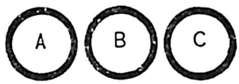

Fig 5.

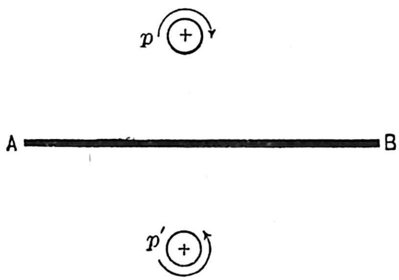

Fig. 6.

meshes. The downward displacement of the positive cells has distorted these meshes, which are normally square. Since this cell structure of the ether is assumed to be elastic, its distortion, as represented in Figs. 3 and 4, represents potential energy. The amount of potential energy per unit volume is proportional to the square of the displacement. This is consistent with the fact that the energy (potential) per unit volume in an electric field is proportional to the square of the field intensity.

The two positive cells to the right of the middle cell in Fig. 4 being displaced downwards, may be conceived to exert troques upon the middle cell, as shown by the arrows *c* and *d*; which troques are proportional to the intensity of the electric field, *i. e.*, to the displacements of the cells. The cells to the left exert equal but opposite troques upon the middle cell. This troque action which accompanies the distortion of the cell structure of the ether is the connecting link between electric field and magnetic field, and it is the basis of the mechanical conception of electro magnetic action.

The Energy Stream in the Electromagnetic Field.—A region in which electric field and magnetic field coexist is called an electro-magnetic field. It has been shown by J. H. Poynting that energy streams through an electromagnetic field in a direction which is at right angles both to the electric field and to the magnetic field at each point, and that the amount of energy per second which streams across one square centimeter of area is proportional to the product of the electric and magnetic field intensities. In case the electric and magnetic fields are not at right angles to each other the energy stream is proportional to the product of the intensities of the two fields into the sine of the included angle.

Explanation of Energy Stream.—Consider three gear wheels, A, B and C, Fig. 5. Let A and C exert equal and opposite torque actions upon B. Then, if the wheels are turned, work will be transmitted from A to C, or from C to A, according to direction of turning and to direction of troque action, and the rate of transmission of work will be proportional to the product of troque action into speed.

Imagine the cells in Fig. 3 to be rotating, positive cells in one direction, negative in the other, about axes perpendicular to the paper. This constitutes a magnetic field perpendicular to the electric field, which is towards the bottom of the page. On account of the troque actions between the cells, as before explained, energy will be transferred to the right (or left) by each chain of geared cells at a rate which is proportional to the product of the intensity of the magnetic field into the intensity of the electric field, and the energy per second flowing across an area perpendicular to both electric field and magnetic field is proportional to the product of the respective field intensities into the area; for this area is proportional to the number of rows of cells which are acting as chains of gear wheels. The energy stream, that is, energy per unit area per second, is therefore proportional to the product of magnetic and electric field intensities, and is at right angles to both.

The Electric Current.—Consider a wire, AB, Fig. 6, along which an electric current is flowing. The magnetic field on opposite sides of AB is in opposite directions, so that positive ether cells at p and p^1 are rotating in opposite directions, as shown. Since an electric current may be maintained for an indefinite time, this opposite rotation of positive ether cells on the two sides of AB cannot be due to an ever-increasing ether distortion (the cells are geared together, as it were), but there must be a slip between adjacent cells somewhere between p and p^1. This slip between adjacent ether cells takes place in the material of the wire, and constitutes the electric current.

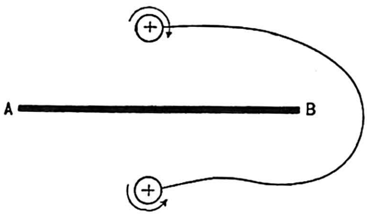

Fig. 7.

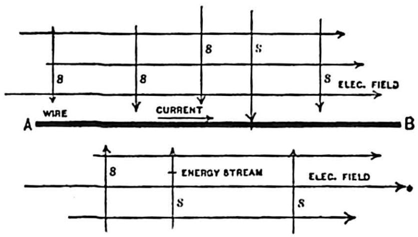

Fig. 8.

Steady electric currents flow in closed circuits. Let AB, Fig. 7, be a wire carrying a steady electric current. If this wire does not form a closed circuit, the opposite rotations of like ether cells on opposite sides of AB cannot continue without adjacent cells slipping on each other somewhere along any chain of cells passing around the end of AB. That is, lines of steady slip of the ether cells are necessarily closed lines. When a current does flow in a circuit which is not closed, and increasing ether distortion (electric field) is produced around the end portions of the circuit, which produces (constitutes) electric charge there, as explained later.

Flow of Energy in the Neighborhood of an Electric Current.—Let Fig. 8 represent the neighborhood of a long wire, AB, through which electric current is flowing. The electric field in the neighborhood is parallel to the wire, and the magnetic field circles around the wire. The product of magnetic field intensity into electric field intensity is the energy stream, and this is directed towards the wire from all sides. This energy streaming in upon the wire changes into heat which appears in the wire. In case the wire is of high resistance, the electric field (volts per centimeter) is of great intensity, and, for a given current (given intensity of magnetic field) the energy stream is correspondingly intense, making the wire very hot. The electric field is not everywhere perpendicular to the wire, especially near the battery or dynamo which is maintaining the current. The energy therefore streams out from the battery or dynamo through the whole region surrounding the wires, and the energy stream turns in upon the wire throughout its length.

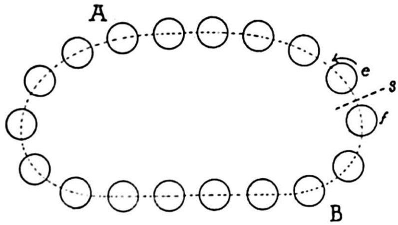

Fig. 9.

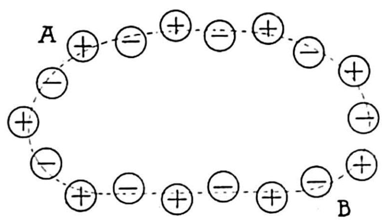

Fig. 10.

The Charge on a Condenser and Its Disappearance when the Condenser Plates are Connected by a Wire.—Consider a closed chain of gear wheels, AB, Fig. 9. If the gears are allowed to slip at any point, *s*, the gear *f* being held stationary and the gear *e* turned in the direction of the arrow, then the chain of gears will be distorted, as shown in Fig. 10. Conversely, a chain of geared wheels, which by elastic action tend to stand in a row,* will be relieved from such a zig-zag distortion, as shown in Fig. 10 by permitting the gears to slip anywhere along the chain and the potential energy stored in the distorted chain will be geared towards the place where slipping takes place.

Let A and B, Fig. 11, be two metal plates and let each dotted line represent a closed chain of ether cells, like Fig. 9. An electric current forced through the wire means the forced slipping of ether cells all along the wire and each chain of the cells, initially like Fig. 9, becomes distorted like Fig. 10.

*The chains of positive and negative ether cells are thought of as standing in zigzag rows when undistorted, as shown by the horizontal rows in Figs. 2 and 3. Hereafter the chains of cells are to be thought of as straight (or uniformly curved) when free from distortion, in order that the diagrams may be simpler.

Throughout the region between A and B the positive ether cells receive an upward displacement and the negative cells receive a downward displacement, that is, this region becomes an electric field and the plates A and B become oppositely charged.

If the charged plates are now connected by a wire, as shown in Fig. 11, each closed chain of geared cells is cut by the wire, slipping begins all along the wire and the energy of each distorted chain of cells is transmitted along the chain, flowing into the wire as indicated by the arrows in the figure.

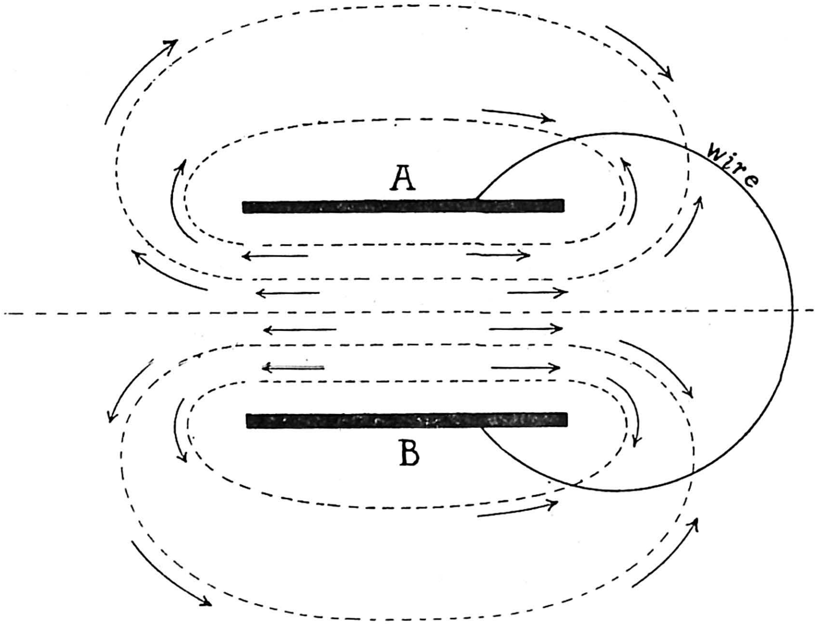

Fig. 11.

An electric spark is a line of slip produced by the breaking down of the mechanism which sustains the electric stress, and the electric energy flows in upon a spark as it does upon a wire carrying current.

The Electric Oscillator.—Let AB, Fig. 12, be the balls of the oscilator upon which electric charge has been collecting. Consider a chain of cells which, when undistorted, lies along a dotted line, which is everywhere perpendicular to the lines of force of the electric field. When A is positively charged, this chain is distorted as shown (in part), but since it is a closed chain, this distortion is fixed. When a spark occurs at the gap, a line of slip is established across this chain, and the distortion disappears as above explained.

If the slip takes place with great friction (high electrical resistance in the gap), the cells at the spark begin turning slowly, and the entire energy of the electric field is geared into the gap and changed to heat. If the slip is almost frictionless (low electrical resistance), the electrical energy is used mostly in overcoming the inertia of the cells as they are set rotating, and after a very short interval of time a very large part of the electrical energy. will have been converted in kinetic energy of the rotating cells (magnetic energy). During this conversion,

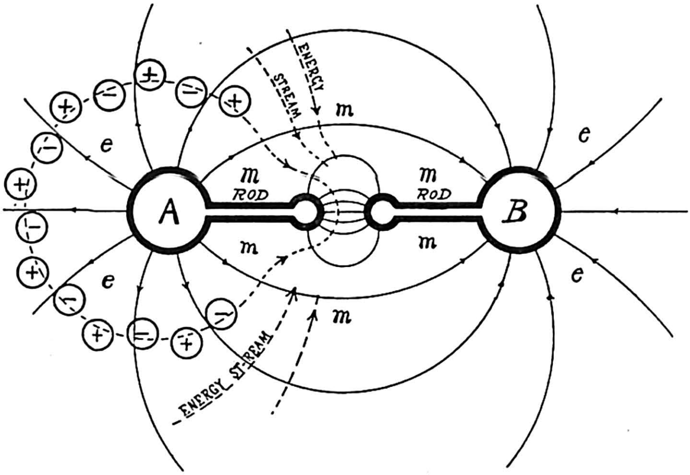

Fig. 12.

energy, streaming along the dotted lines, largely disappears from the regions *ee*, and is distributed mainly in the region *mm*. When the chain of cells has been freed from distortion, the rotatory motion of the cells between A and B will have reached a maximum, and on account of their momenta the cells will continue turning and produce a distortion of the chain in a reversed sense. At the same time the energy will, to a large extent, stream back from the region *mm* to the region *ee*, the ball A will be negatively charged, and the ball B will be positively charged. The reversed distortion of the chain of cells is then relieved by a reversed slip (a reversed current in the rods and gap), and so on.

These oscillatory changes take place so rapidly that the portions of the distorted ether which are remote from AB do not follow the changes promptly. This gives rise to electrical waves.

The Electromagnetic Wave.—Before describing the electromagnetic wave, it will be instructive to consider a common property of all wave motion. Take, for example, a wave traveling over the surface of water. This wave consists of a moving hill of water, and a given particle of water is set in motion when the wave reaches it and comes immediately to rest when the wave has passed by. What is it that supports the hill of water and where is the unbalanced force which causes the water to gain velocity and then lose it again as the wave passes by? The fact is that a wave always consists of two elements, a local distortion of the medium and a local acceleration of the medium and the forces which are associated with the distortion are precisely the forces which produce the acceleration. The distortion sustains the acceleration, as it were, and the acceleration sustains the distortion as they both travel along together. The two are mutually dependent. Everyone knows, furthermore, that to lift an oar from the water leaving a depression, produces two waves, one traveling forwards and the other backwards, and that each of these waves has a motional phase and a distortional phase, although the original disturbance was a depression (distortion) pure and simple, without any motional phase. Similarly, a sharp blow on a long stretched wire imparts motion to a short portion of the wire, and this motion quickly generates two oppositely moving waves each having a motional phase and a distortional phase, although the original disturbance consisted of motion only.

An electromagnetic wave also consists of a state of distortion and a state of motion traveling along together and mutually sustaining each other. The distortion is electric field and the motion is magnetic field. A layer of electric field unsustained breaks up into two electromagnetic waves just as an unsustained bend in a stretched wire breaks up into two waves, and a layer of magnetic field unsustained breaks up into two oppositely-moving electromagnetic waves just as a local state of motion of a wire produced by a hammer blow breaks up into two waves.

Some detailed ideas of the constitution of an electromagnetic wave may be obtained with the help of Fig. 13, in which the fine arrow lines represent the lines of force of a layer of electric field and the fine dots represents the lines of force of a layer of magnetic field directed towards the reader. This layer of electric and magnetic field moves to the right and constitutes an electromagnetic wave. The mutual sustaining action of the electric and magnetic field is explained in terms of Maxwell's cellular ether theory as follows: The two wires should be thought of as two broad metal ribbons for the sake of simplicity and they bound the electric wave very much as a speaking tube

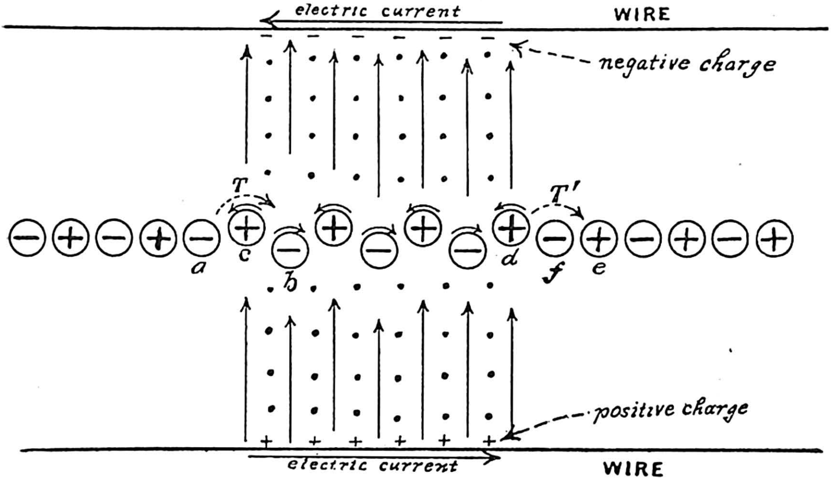

Fig. 13.

bounds a sound wave which passes through it. Midway between the two wires which bound the wave a simple chain of ether cells is indcated in Fig. 13. The part of this chain which lies within the wave is distorted and this distortion represents the electric field. Furthermore, those cells which lie within the wave are rotating as indicated by the small curved arrows, and this rotation represents the magnetic field.

Throughout the middle portions of the wave each rotating cell is acted upon by equal and opposite torques by the adjacent cells ahead of it and behind it, as explained in connection with Fig. 4. Therefore all the cells in the middle portion of the wave rotate at unchanging speed. The cell *d* however, exerts an unbalanced torque upon cell f as indicated by a dotted arrow T^1 and this torque quickly sets the cell f rotating, while the cell b exerts an unbalanced torque T upon the cell c which quickly stops the rotation of the cell c. Thus the state of motion of the cells between c and f travels to the right. Likewise the state of distortion travels to the right inasmuch as this state of distortion is inseparably associated with the torque actions just referred to.

The terminating of the electric lines of force at the wires which bound the wave constitutes electric charges, positive on one wire and negative on the other, and electric currents flow along the two wires within the space occupied by the wave. This electric current flows in one direction in one wire and in the opposite direction in the other wire. The necessity for this current flow is involved in the bounding of the wave by the metal wires or ribbons, inasmuch as the rotation of the ether cells on one side of the ribbon with stationary ether cells on the other side, requires slipping of adjacent cells in the material of the ribbon.

The mutual dependence of the moving electric and magnetic fields which together constitute an electromagnetic wave may be expressed in terms more or less familiar. Before attempting this, however, it may be well to state that the fundamental relation between force and acceleration in mechanics *is in electromagnetic theory the same thing as the relation between electromotive force and changing magnetic flux.* A magnetic field of intensity H moving sidewise at a velocity v produces electromotive force according to the equation $\mathrm{E} = \mathrm{l\ H\ v}$, or in general $\mathrm{E} = \mathrm{l}\ \mu\ \mathrm{H\ v}$ or since E/l is electric field intensity e we may write

$$\mathrm{e} = \mu\ \mathrm{H\ v} \quad (1)$$

in which μ is the magnetic permeability of the medium.

Now, another fact, not generally known among practicing electricians, is that an electric field of intensity e moving sidewise at velocity v produces magnetomotive force F according to the equation $\mathrm{F} = \mathrm{l\ e\ v}$, or in general $\mathrm{F} = \mathrm{l\ k\ e\ v}$, or, since F/l is magnetic field intensity H, we may write,

$$\mathrm{H} = \mathrm{k\ e\ v}. \quad (2)$$

in which k is the inductivity of the dielectric medium.

The condition which must be satisfied in order that the whole of H may be due to the sidewise motion of e and at the same time the whole of e due to the sidewise motion of H, that is, that H and e may sustain each other completely, is that equations (1) and (2) be simultaneous equations. Treating equations (1) and (2) therefore as simultaneous equations we have:

$$v = \frac{1}{\sqrt{\mu k}} \qquad (3)$$

and

$$\tfrac{1}{2}\mu H^2 = \tfrac{1}{2} k e^2 \qquad (4)$$

Equation (3) gives the velocity at which H and e must move sidewise so as to be mutually dependent on each other. This is, of course, the velocity of an electromagnetic wave. Equation (4) expresses the fact in a so-called pure electromagnetic wave the magnetic energy is always and everywhere equal to the electric energy.

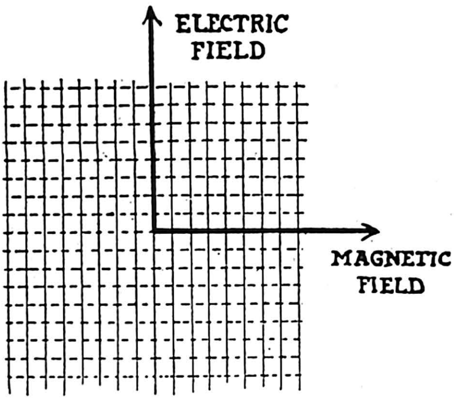

Fig. 14.

Fig. 14 represents a front view of a plane electromagnetic wave approaching the reader. The full lines represent electric lines of force and the dotted lines represent magnetic lines of force.

Wave Distortion.—The distortion of an electromagnetic wave, which is so troublesome in telephonic transmission, is due to the resistance of the wires or ribbons which bound the waves and to the conductivity of the insulating medium between the wires, and a clear understanding of electromagnetic wave distortion depends upon clear ideas of these effects of line resistance and line leakage.

One may see from Fig. 13 that the frictional resistance of the slipping ether cells in the material of the wires (the mechanical analogue of line resistance) causes the angular velocity of the ether cells (the mechanical analogue of magnetic field) to decrease continually. In fact the effect of line resistence is to cause the magnetic field H in an advancing electromagnetic wave to decrease as the wave moves along, while the electric field in an advancing wave is not directly affected by line resistance.

On the other hand, if the structure of the medium in the region occupied by the advancing wave gives way continuously to the ether stress which constitutes the electric field, this ether stress will decrease steadily as the wave advances just as a state of stress in a stretched molasses-candy rope decreases steadily with time. This steady giving way of the ether under stress is the mechanical conception of the conductivity of the insulating medium between the wires, that is of line leakage, and in fact the conductivity of the insulating medium between the wires of a transmission line causes the electric field to die away as the wave moves along, while the magnetic field in an advancing wave is not directly affected by line leakage.

The distortion of an electromagnetic wave pulse as it travels along a transmission line is due wholly to the fact that line resistance reduces magnetic field only and that line leakage reduces electric field only, and a detailed description of wave distortion depends upon a clear understanding of an impure wave, as it is called, that is, a wave in which either the magnetic field has been reduced below the value which is necessary to sustain the electric field or in which the electric field has been reduced below the value which is necessary to sustain the magnetic field.

It has already been pointed out how a local bend in a stretched string which is unsustained by local acceleration breaks up into two oppositely moving pure waves and how a local movement of a stretched string due to a hammer blow breaks up into two oppositely moving pure waves. From this it is not difficult to see that if a wave, originally pure, were to travel along a string which is strung through a viscous liquid, the viscosity of the liquid would cause the motion to die away as the waves travel along but would not directly affect the distortion of the string, and therefore, at each point on the string, that part of the distortion which is sustained by what remains of the motion, travels onward as a pure wave, while that part of the distortion which is not sustained, the excess of distortion as it were, breaks up into two oppositely moving pure waves, one of which merges with what Heaviside calls the head of the wave, and the other moves backwards and generates what Heaviside calls the tail of the wave.

Fig. 15 shows in an exaggerated way, an electromagnetic wave in which the magnetic field M has been reduced by line resistance, while the electric field E has not been reduced, and

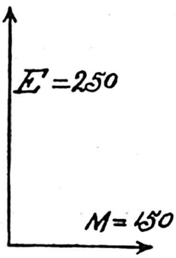

Fig. 15.

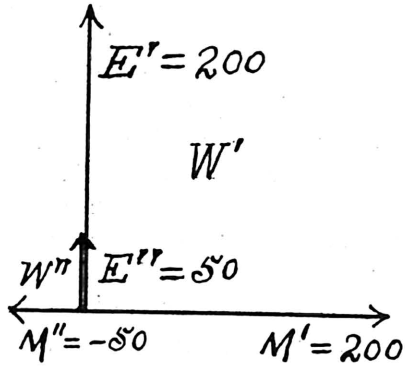

Fig. 16.

Fig. 16 shows the two pure waves W′ (which consists of E′ = 200 and M′ = 200) and W″ (which consists of E″ = 50 and M′ = — 50) into which this so-called impure wave resolves itself. The wave W′ continues to move forward and the wave W″ moves backwards.

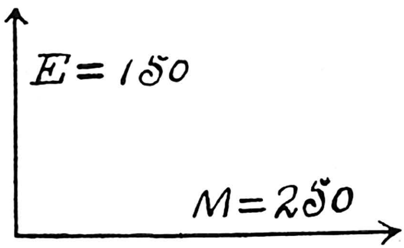

Fig. 17.

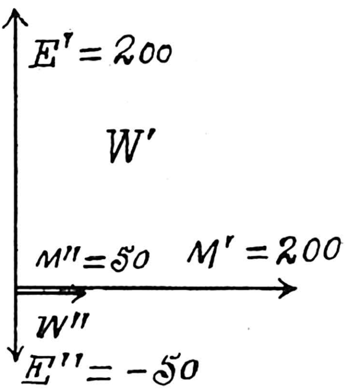

Fig. 18.

Similarly Fig. 17 shows an electromagnetic wave in which the electric field E has been reduced by line leakage, and Fig. 18 shows the two pure waves W′ and W″ into which this impure wave resolves itself.

Before describing the actual process of wave distortion, it is necessary to adopt a scheme for representing an electromagnetic wave pulse graphically. Such a representation is indi-

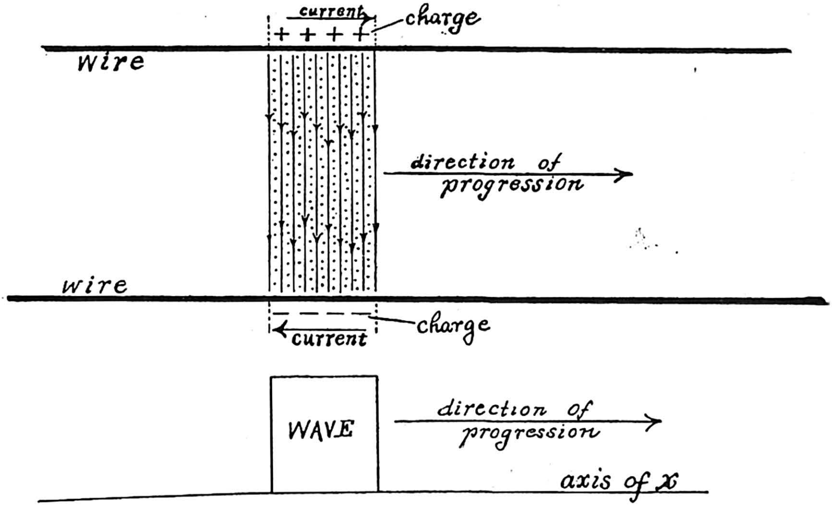

Fig. 19.

cated in Fig. 19. The rectangular curve marked "*wave*" in the lower part of this figure represents the pure wave pulse which is shown directly above it.

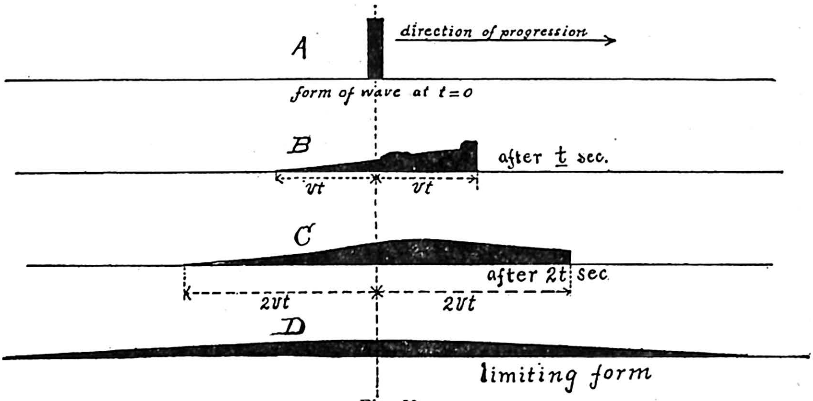

Fig. 20.

The black rectangle A, Fig. 20, represents a pure electromagnetic wave which at a given instant is supposed to be called into existence out on an indefinitely long pair of wires. As this wave proceeds its magnetic field is decreased by line resistance (or its electric field is decreased by line leakage) and the wave becomes impure. The result is, as before explained, that a portion of the energy of this original wave begins to shoot back ward at every point along the line as a reflected wave. This reflected wave, which is of course in its turn partially re-reflected as it travels backwards, generates the tail of the wave after t seconds is shown by B. The head of this resulting wave has been reduced in values of E and M and the tail reaches to a distance vt back of the original position of the wave. The form of the wave after $2t$ seconds is shown by C and the final limiting form of the wave is shown by D. The fatness of the tail is greatly exaggerated in Fig. 20.

When, during an element of time, an electromagnetic wave has suffered decrements of E and M represented by $\triangle$ E and $\triangle$ M, due to line leakage and line resistance respectively (both E and M being expressed in such units that their numerical value will be equal when $\frac{1}{2}\mu M^2$ is equal to $\frac{1}{2} k E^2$), then the common numerical value of E and M in the reflected wave which shoots out into the tail is $\frac{1}{2}(\triangle E - \triangle M)$ and the residual numerical value common to both E and M in the head of the wave is $E - \frac{1}{2}(\triangle E - \triangle M)$ or $M - \frac{1}{2}(\triangle E - \triangle M)$. The loss of energy in the head of the wave due to line leakage is $k E \triangle E$, the loss of energy in the head of the wave due to line resistance is $\mu M \triangle M$, and the loss of energy from the head due to the spreading into the tail of the wave is $\frac{1}{8}(k+\mu)(\triangle E - \triangle M)^2$. Therefore when $(\triangle E - \triangle M)$ is small the loss of energy from the head of the wave by line leakage and by resistance is vastly larger than the loss of energy by the spreading of energy into the tail. When, however, $(\triangle E - \triangle M)$ is large the loss of energy from the head of the wave by the spreading of energy into the tail is of the same order of magnitude as the loss of energy due to line leakage or to line resistance.

When $(\triangle E - \triangle M)$, in previous section on Wave Distortion, is small the spreading of energy into the tail is negligible in comparison with resistance loss and leakage loss, so that when $(\triangle E - \triangle M)$ is small any increase of line resistance or leakage with a view of still further eliminating wave distortion produces an increased loss of energy from the head of the wave, which is nearly equal to the increase of resistance loss or leakage loss of energy because when ($\triangle$E—$\triangle$M) is small the loss of energy from the head of the wave is sensibly equal to the sum of the resistance and leakage losses.

On the other hand when ($\triangle$E—$\triangle$M) is large the spreading of energy into the tail is considerable, and any increase of loss resistance or leakage with a view of largely eliminating wave distortion produces of course an increased loss of energy from the head of the wave, but this increased loss from the head of the wave is much less than the increase of resistance loss or leakage loss, because the spreading of energy into the tail of the wave is greatly reduced by the increase of line resistance or line leakage.*

On all ordinary telephone lines and cables the loss of energy due to line resistance exceeds the loss due to line leakage, except perhaps when the line or cable is out of order. Therefore, an increase of line leakage generally reduces wave distortion and increases the distinctness of telephonic transmission. Whether this increase of line leakage is justifiable depends largely upon whether it is constant and independent of the weather, upon the degree of wave distortion, for if the wave distortion is excessive the reduction of useful energy in the head of the wave is by no means equal to the increased loss of energy by leakage, as explained above, and upon the extent to which the diffused energy that follows in the tail of every wave pulse seriously interferes with the distinctness of the telephonic transmission.

Reduction of Line Losses and of Wave Distortion by Increasing Line Induction.—The use of increased inductance for decreasing wave distortion and for decreasing the loss of energy in the

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*A statement was given in the *Electrical Review*, Vol. 46, p. 242, Feb. 11, 1905, in illustration of this tendency to actually conserve the useful energy in a wave pulse by increese of line resistance or line leakage. This statement was, that an increase of the leakage of a perfectly insulated line so as to make the leakage loss equal to the resistance loss results in an increased energy loss from the head of the wave in the ratio of $1\frac{1}{2}$ to 2. This statement is faulty inasmuch as for some inexcusable reason it was based upon the aberrant idea that the energy of a wave is proportional to E (or M) instead of to E^2 (or M^2) as everyone knows.

head of a wave pulse while traveling over a given line may be shown in a very simple and elementary way as follows:

In the first place equations (3) and (4) may be written

$$v = \frac{1}{\sqrt{LC}} \qquad (5)$$

$$\tfrac{1}{2} L i^2 = \tfrac{1}{2} CE^2 \qquad (6)$$

in which L is the inductance of the transmission line per unit length, C is its capacity per unit length, i is the current flowing outward in one line and backward in the other at a given point on the line, and E is the electromotive force between the lines at the given point. The derivation of (5) and (6) from (3) and (4) need not be considered, although it is simple.

Let us consider a special case for the sake of clearness. Suppose a battery of electromotive force E is suddenly connected to the sending end of the line and a moment later disconncted. A rectangular wave of length l would be started along the line. This wave contains a certain amount of energy, a certain portion of which is dissipated or left dangling along the line, and the remainder arrives in the head of the wave at the receiving station. Suppose now that L is quadrupled, C and E being kept unchanged for the sake of comparison. Then from equations (5) and (6) v would be halved and i would be halved. If the sending battery is now connected to the line for the same length of time as before, a rectangular wave of length $\frac{1}{2}\,l$ will be started along the line, inasmuch as v is halved. Now, current flows in the line wires only where the wires bound the wave, that is in length l of the double line in the first case and in length $\frac{1}{2}\,l$ of the double line in the second case. Furthermore, the time required for the wave to reach the distant end of the line is twice as great in the second case. Therefore R i^2 t, or the total energy dissipated in the wires from the head of the wave, is one-quarter as great in the second case as in the first, since R is halved, i^2 is quartered, and t is doubled. But the total energy is the wave is half as great in the second case as in the first case, since l is halved and the energy per unit length is unchanged, viz.: ($\frac{1}{2}$ L i^2 + $\frac{1}{2}$ CE^2. Therefore the percentage loss of energy from the head of the wave is halved by quadrupling the line inductance. In this example resistance loss alone is considered inasmuch as leakage loss is constant under the assumed conditions. The effect of increasing the inductance of a telephone line, loading the line as it is called, is the same as decreasing its resistance insofar as line losses and wave distortion are concerned, provided we are concerned with pure wave transmission. It must be remembered indeed that equations (4) and (6) apply to pure waves only. As applied to pure waves these equations indicate that for a given line there is a fixed relation between voltage and current. Neither can be changed without changing the other unless the line constants are changed. The inapplicability of equations (4) and (6) in their simple form to an alternating current transmission line is at once evident when we consider that there is no fixed relation between current and voltage on such a line. Alternating current transmission is not pure wave transmission.

The extension of this discussion to waves produced by steadily maintained periodic electromotive forces is beyond the scope of the present lecture, and the limitation of the discussion to the wave pulse is deemed advisable for two reasons,

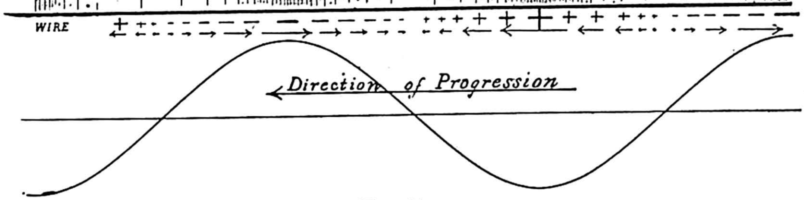

Fig. 21.

+ and — signs represent electric charge on the wires.
The short arrows represent electric current.
Dots represent magnetic lines of force.
Fine lines represent electric lines of force.

E.m.f. between wires at point x equals E Sin $\left(\frac{x}{\gamma}+\frac{t}{\tau}\right)$

Current in one wire at point x equals I Sin $\left(\frac{x}{\gamma}+\frac{t}{\tau}\right)$

viz.: (a) the theory of wave distortion may be completely covered by a consideration of the behavior of a wave pulse, and (b) the physics of wave motion in a medium involving reactions, which in optics gives rise to absorption and dispersion and in electric transmission gives rise to distortion and attenuation, is vastly simpler for the wave pulse than for the wave train. The wave train represents a steady state of a system under the action of a periodic force which has been kept up long enough for the system to settle down to a steady state of motion, and *this steady state depends upon what is taking place at every point in the system*, whereas the wave pulse, the head at any rate, has to do only with what takes place at one single point in the system. The physical nature of a simple electromagnetic wave train is shown in Fig. 21. The curved line at the bottom of the figure is a representation of the wave train.

The reader should keep in mind the fact that all of the discussion as outlined above is based first upon the determination of electromotive force, and current relations on the assumption that line resistance and line leakake are zero, after which line resistance and line leakage are considered as disturbing effects.

It is entirely misleading to the learner to dwell upon the limitations of Maxwell's cellular theory of the ether. There is scarcely a phenomenon of electromagnetic action which is not very greatly simplified by the use of these conceptions. The choice is, as I indicated at the opening of my lecture, between the use of differential equations, not for final integral solutions, but for descriptive purposes, as it were, or the use of mechanical conceptions. In fact, every phase of the foregoing discussion might be easily expressed in differential notations. It is, however, needless to say that such mode of expression would be to a great extent ineffectual. The difficulty with the ordinary mechanical notions which are so fundamentally fixed in electricians' minds, is that they do not give any insight into electromagnetic action, and the difficulty with the cellular theory, aside from the one mentioned by Poynting, is that it may perhaps never give any insight into the phenomena of moving electrons. My advice to the younger readers is to swallow Maxwell's cellular theory and digest it, and do not turn away from it in disgust because some one says, truly, that is does not contain the secret of the universe.